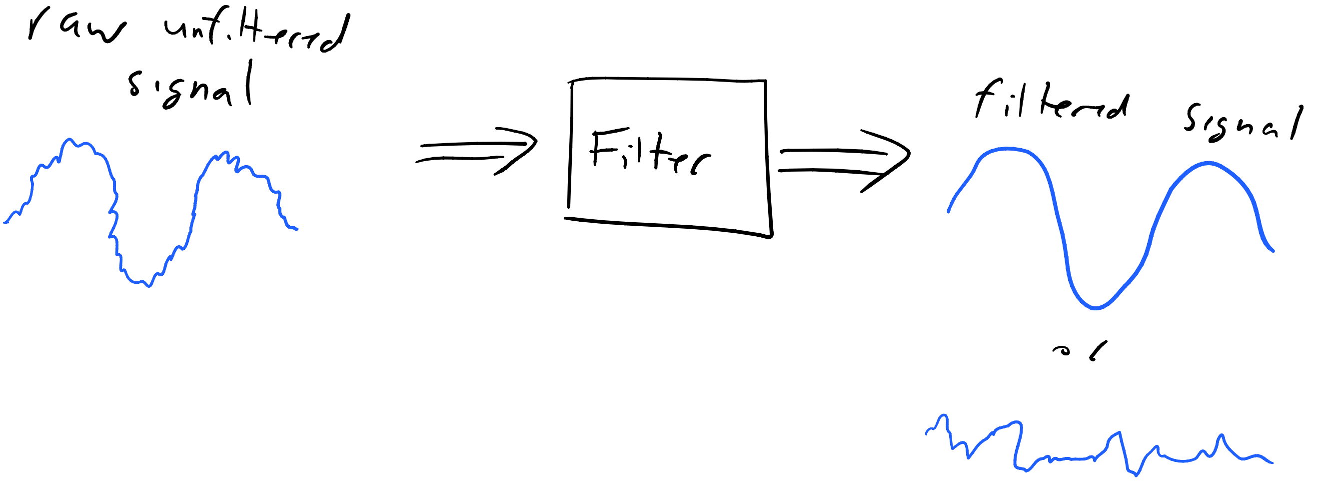
Filtering

Sunday, May 8, 2016 8:02 AM

# Filtering

We've talked about how to protect our systems from noise, but we still need to reduce unwanted parts of our signals. That's what filters do.

Filters remove unwanted frequency components from a signal and/or enhance wanted components.



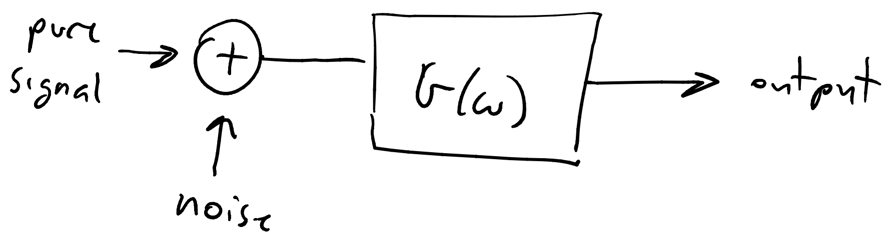

## There are two types of filters

* Analog filters - use analog components to filter a signal, including resistors, capacitors, inductors, and operational amplifiers.

* Digital filters – filter a signal mathematically inside a processor, be it a general purpose PC or special digital signal processor (DSP).

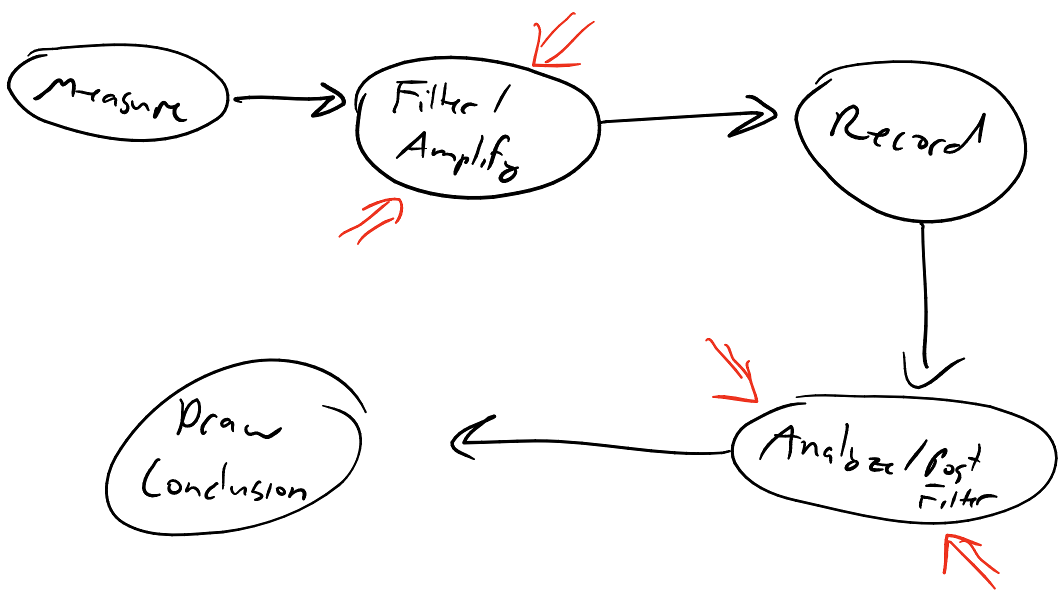
## Common Filtering Scenario

We are measuring something with a system that has an imperfect response $G(\omega)$ with noise superimposed.




Ideally we need to remove the noise and system response to reconstruct the real pure signal.

## Typical Workflow



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Why filter before recording? Just record everything and filter in post processing.

* Information about the signal is lost during digitization and recording

* Wide band noise on a slowly varying signal would require a high digitization rate

* Wide band noise on a narrow band signal will greatly reduce the signal-to-noise ratio

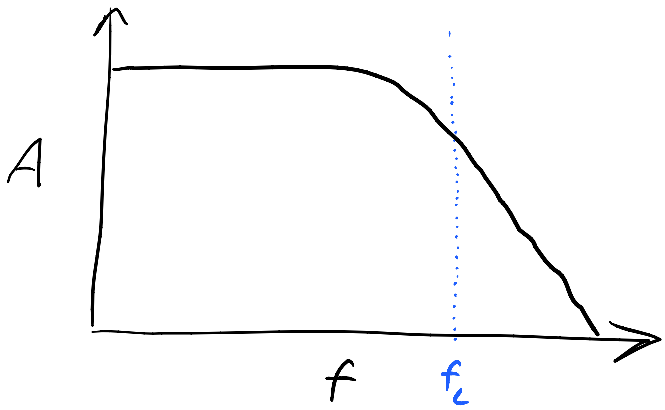
## Rules of Filtering (according to me)

1. Do no harm. If you don't understand what you are doing, stop and learn.
2. Filter out known noise sources (i.e. 60Hz AC) only if there is no expected signal at that frequency
3. Filter systematics & noise that degrade system response
4. Remember the Nyquist frequency and never, ever, ever approach it.
5. Filter systems to . .

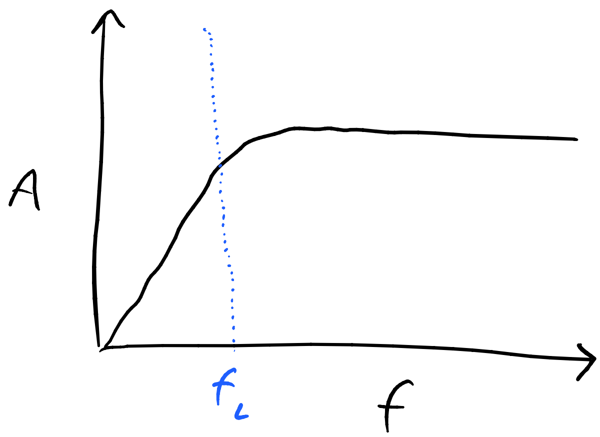
5. Filter systems to only respond over their linear "dynamic range"

## Types of Filters

Low pass - passes frequencies below some cutoff $f_c$

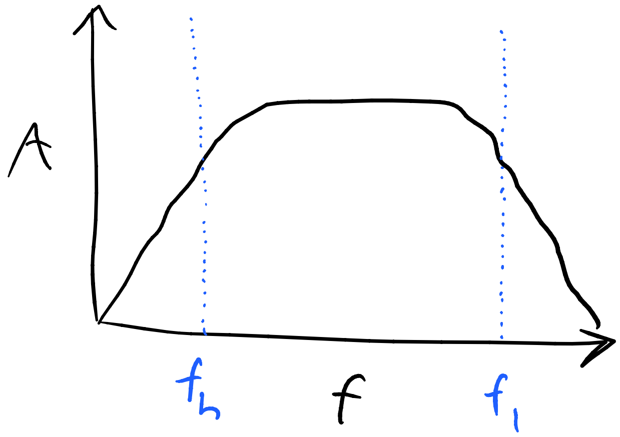



High pass - passes frequencies above some cutoff $f_c$

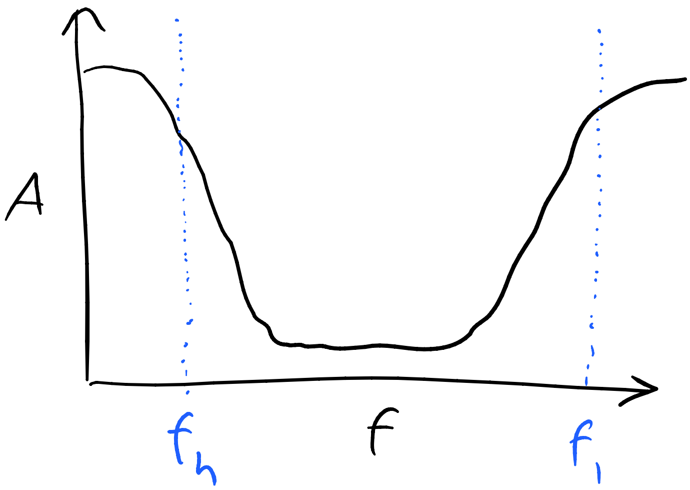



Bandpass - passes frequencies between a highpass cutoff $f_h$ and lowpass cutoff $f_l$

$f_h$ and lowpass cutoff $f_l$



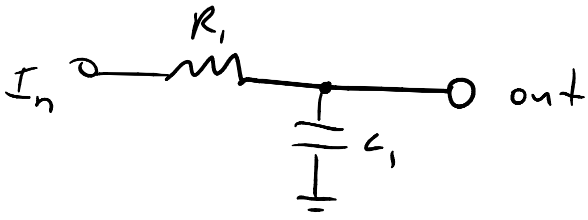

Bandstop - passes frequencies outside of the region between $f_h$ and $f_l$




* Narrow band stop filters are often called notch filters.

## Simple Single-Pole Passive Filters

Low Pass




Let's analyze this:

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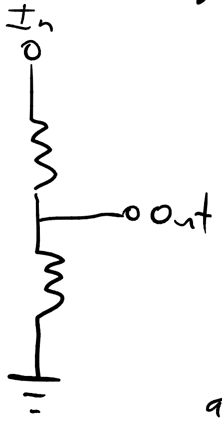



$$V_{out} = V_{in} \cdot \frac{R_2}{R_1 + R_2}$$

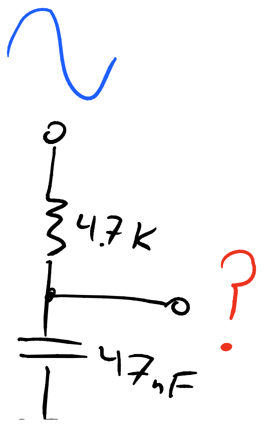
But we learned capacitors have a frequency dependent impedance.

$$X_c = \frac{1}{2\pi f C} \qquad Z = \sqrt{R^2 + X_c^2}$$

$$V_{out} = V_{in} \cdot \frac{X_c}{\sqrt{R^2 + X_c^2}} = V_{in} \frac{X_c}{Z}$$

## Activity

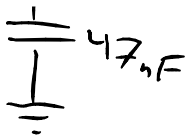
Calculate Vout of this filter for a 10V sinusoidal input with a 10V amplitude at 100 Hz and 10 kHz




@ 100 Hz

$$X_c = \frac{1}{2\pi f C} = 33863\,\Omega$$

$$V_{out} = V_{in} \frac{X_c}{\quad}$$




$$V_{out} = V_{in} \frac{X_C}{\sqrt{R^2 + X_C^2}} = 9.9\,V$$

@ 10 kHz

$$X_C = \frac{1}{2\pi f C} = 338.6\,\Omega$$

$$V_{out} = V_{in} \frac{X_C}{\sqrt{R^2 + X_C^2}} = 0.718\,V$$

* The cutoff frequency is where the output voltage is $\frac{1}{\sqrt{2}}$ of the input.

$$f_c = \frac{1}{2\pi R C}$$

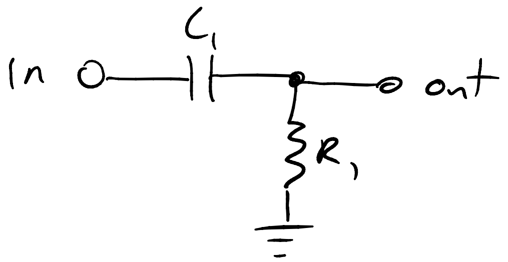
For the example above, what is $f_c$?

$$f_c = 720\,Hz$$

* Phase shift $\phi = -\arctan(2\pi f R C)$

* The falloff for this filter is 20 dB/decade or 6 dB/octave

## High pass

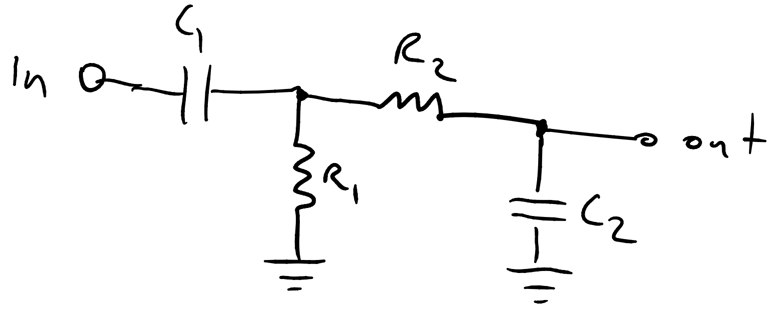



$$V_{out} = V_{in} \frac{R}{Z}$$

* Looks similar to the differentiator we talked about, because it is!

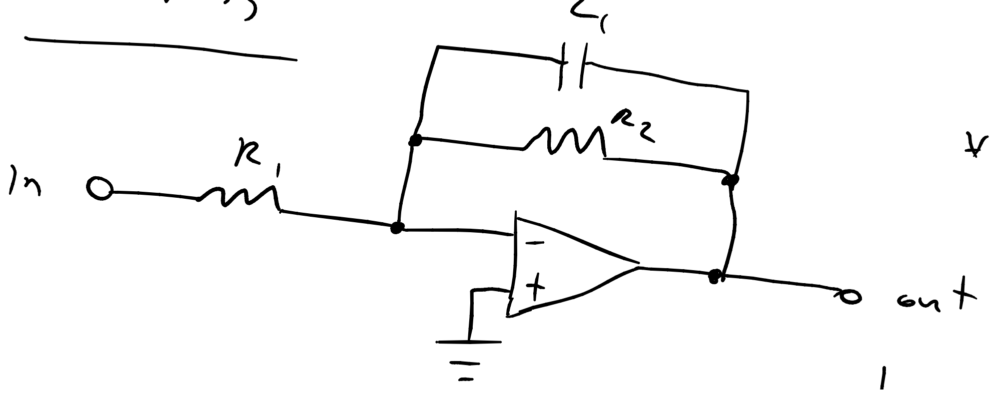
## Band pass

Combine the low & high pass filters!




## Basic Active Filters

### Low Pass




* Inverting amplifier!

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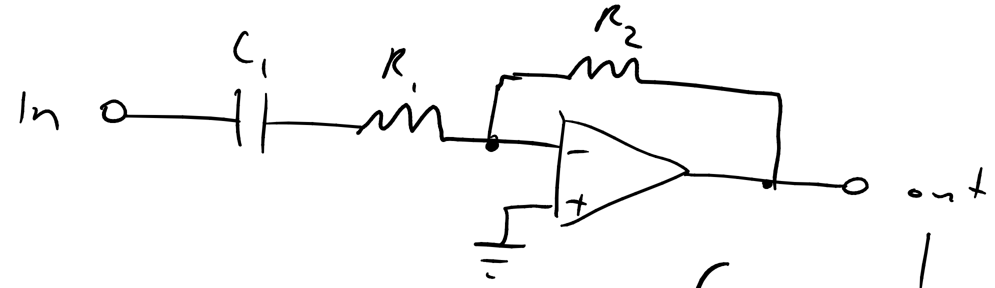
$$f_c = \frac{1}{2\pi R_2 C_1}$$

$$\text{Gain} = \frac{R_2}{R_1}$$

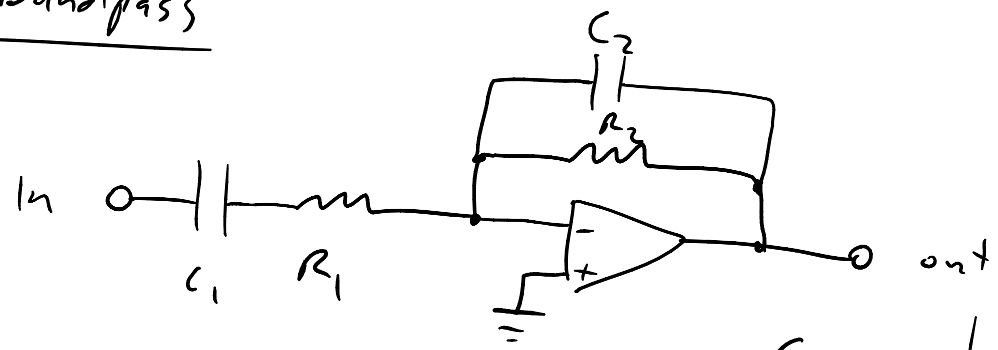
## High Pass




$$f_c = \frac{1}{2\pi R_1 C_1}$$

$$G = \frac{R_2}{R_1}$$

## Bandpass



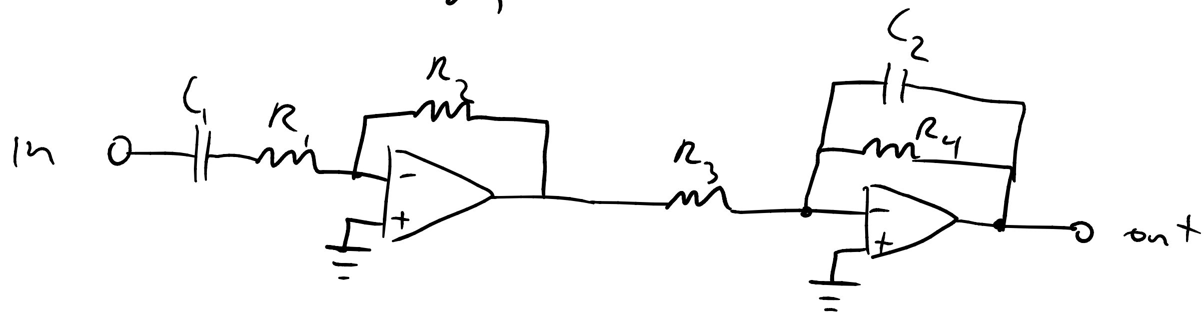

$$f_1 = \frac{1}{2\pi R_2 C_2}$$

$$f_2 = \frac{1}{2\pi R_1 C_1}$$

$$G = \frac{R_2}{R_1}$$

* Can be tricky to solve
  3 eqns w/ 4 unknowns
* More compact design

Daisy Chain Design



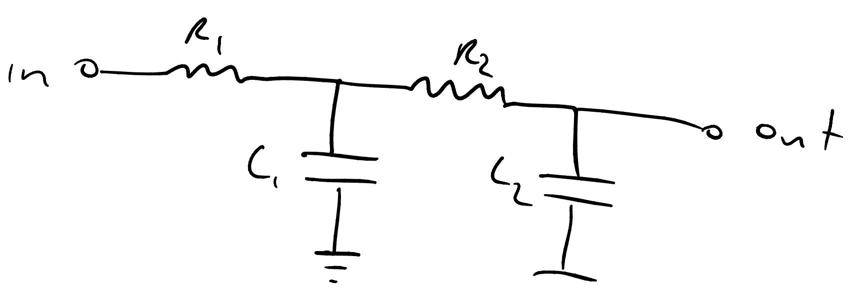

$$f_1 = \frac{1}{2\pi R_1 C_1} \qquad f_2 = \frac{1}{2\pi R_4 C_2}$$

$$G_1 = \frac{R_2}{R_1} \qquad G_2 = \frac{R_4}{R_3}$$

## Higher Order Filters

You can create filters with a sharper fall off by chaining or using more complex designs

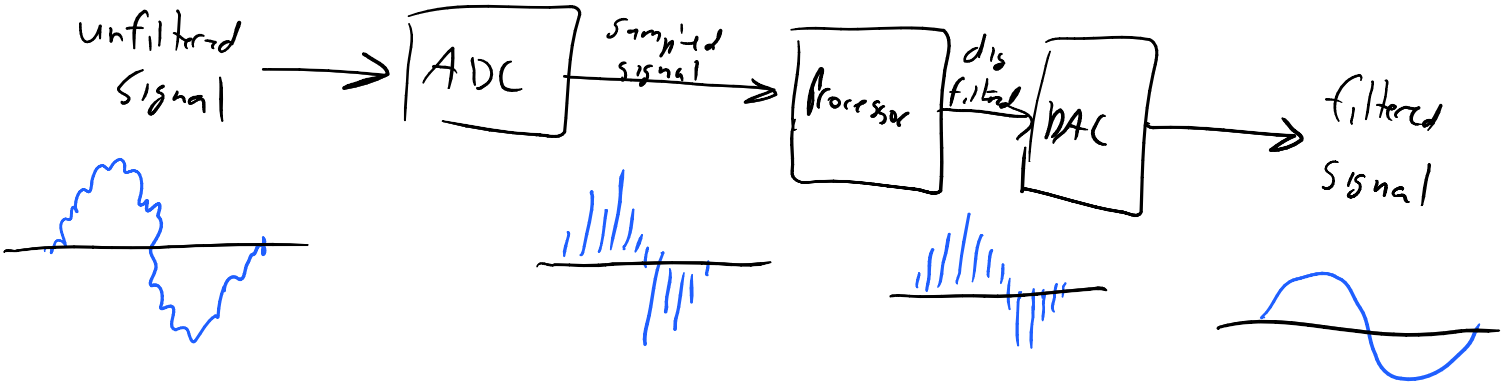
### 2nd order lowpass




$$f_c = \frac{1}{2\pi\sqrt{\;}}$$

$$f_c = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}}$$

## Digital Filters




### Benefits of digital filtering

1) Programmable - easy to change without modifying the circuit design

2) Easy to design, test, and implement. You just need a laptop!

3) Stable with time and temperature - not subject to drift like analog components.

4) Very good low frequency capabilities compared to analog electronics. RF side becoming possible too with adv.

...ting. RF side becoming possible too with advances in DSP design

5) Versatility - can be made adaptive to the signal or situation.

6) Simple and compact - complex filter chains can be made with a relatively small and straightforward hardware design

## Filter Order

* The order of a digital filter is the number of previous inputs used to calculate the current output

Zero order: unity gain, non-unity gain

First order: pure delay filter, two term difference, two term average

Second order: three term average, central difference

## Filter Recursiveness

⇒ non-recursive fil...

⇒ non-recursive filters calculate their output based only on the current and previous input values.

- Also known as a finite impulse response (FIR) filter

⇒ recursive filters calculate their output using inputs in addition to previous output values.

- Also known as an infinite impulse response (IIR) filter

- Recursive generally require fewer terms to get the same response as a non-recursive filter.

* The order of a recursive filter is the largest number of previous input or output values required to compute the current output.